Throughout class today, we refer to the following examples:

**Ex. 1**

![Graph](image1)

**Ex. 2**

![Graph](image2)

If we have a path (or walk) that starts at \( u \) and ends at \( v \),

\[
(u, v) - \text{path} \\
(u, v) - \text{walk}
\]

**Proposition**

Let \( G \) be a graph and \( u, v \in V(G) \). If there is a \((u,v)\)-walk in \( G \), then there is a \((u,v)\)-path in \( G \).

**Proof**

\[
W = (u, \ldots, \ldots, \ldots, v)
\]

Suppose a vertex \( x \) is repeated

\[
W = (u, \ldots, ?, x, ?, \ldots, ? , x, ?, \ldots, v)
\]

\[
W = (u, \ldots, ? , x, x, ?, \ldots, v)
\]

Repeat until no repeated vertices remain.

We have a \((u, v)\)-path.

**Examples**

Consider \((1, 4)\)-walk
(Ex 1)  
(1, 9, 2, 5, 3, 9, 2, 4)  
  
(1, 9, 2, 4)  
  
is a (1, 4)-path

Connected - To
Let G be a graph and u, v ∈ V(G).
We say u is connected-to v if there is a (u,v)-path in G.

Examples:  
1, 4 are connected  
1, 2 are connected  
3, 3 is connected  
4, 8 are not connected  
7, 9 are not connected

Properties  
Let u, v, w ∈ V(G) be vertices  
1. u is connected to u (reflexive)  
2. If u is connected-to v then  
v is connected-to u. (symmetric)  
3. If u is connected-to v and v is connected-to w  
then u is connected-to w. (transitive)

Connected  
A graph G is called connected if for all pairs of vertices (x, y) ∈ V(G), x is connected-to y.

Examples  
Ex 1 is not connected  
Ex 2 is connected
Components (Connected Components)
A component \( C \) of a graph \( G \) is a subgraph of \( G \) that is:
1) \( C \) is connected
2) \( \forall x \in C \), there is no vertex \( y \in G - C \) such that \( x \sim y \).

Cut Vertex/Edge
Let \( G \) be a graph.
\( v \in V(G) \) is a cut vertex of \( G \) if \( G - v \) has more components than \( G \).
\( e \in E(G) \) is a cut edge of \( G \) if \( G - e \) has more components than \( G \).

Cycle
A cycle is a walk of at least length three, where the first and last vertex are the same and no other vertices are repeated.

Examples:
(Ex 1) \((1, 2, 9, 1)\) is a cycle
(1, 9, 3, 5, 2, 4, 6, 1) is a cycle
(1, 2, 4) is not a cycle
(1, 2, 1) is not a cycle

Ex 2 has no cycles.

Tree
A tree is a connected, acyclic graph.

Examples

Ex 1 is not a tree
Ex 2 is a tree.