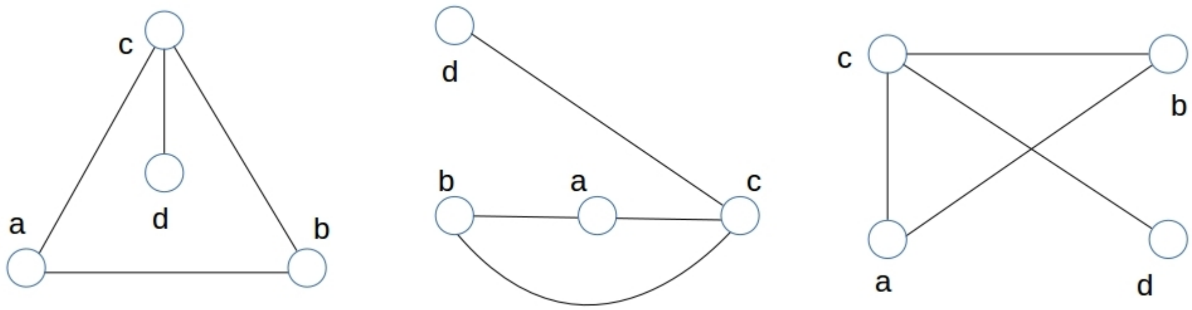
Graph Isomorphism

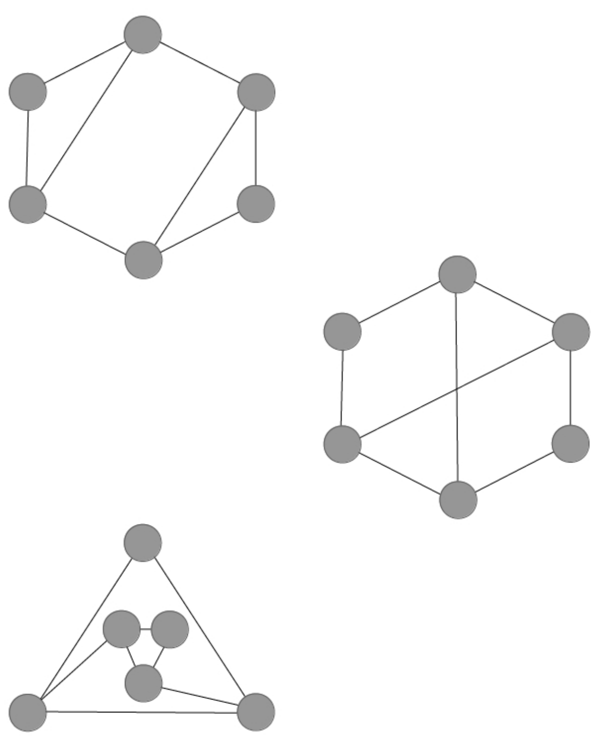
Suppose we are given two graphs, both on the same number of vertices with the same number of edges. Sometimes it is extremely useful to know if they are actually the same graph – for example, if the graphs represent molecular structures, it is useful to know if they are actually the same compound. If they are then any property know to be held by one (for example, specific gravity) will be shared by the other.



This figure, repeated from above, shows three graphs that are all the same graph: they have the same vertices and the same edges. But suppose we threw away the labels, or suppose we used different labels on one of the drawings. Would we still be able to say the graphs were related?

We say that two graphs are **isomorphic** if we can label the vertices in the two graphs with the same labels in such a way that all the adjacencies are preserved: each pair of vertices are adjacent in one graph if and only if they are adjacent in the other graph. The notation for isomorphism is $G \cong H$

As an exercise, consider these three graphs. Each has six vertices and eight edges. Are any of them isomorphic to each other?



Graph isomorphism is not only a practical problem – it is also of enormous theoretical interest. We have seen questions that are easy to answer (such as “Is G Eulerian?”) and questions that are extremely difficult to answer (such as “Is G Hamiltonian?”). The question “Are G_1 and G_2 isomorphic?” is unusual in that it seems to float somewhere between these two extremes ... but nobody knows for sure. It is entirely possible that somebody will discover an easy way to answer this question, but it is also possible that somebody will prove that there is no easy solution. Here again we find ourselves asking simple questions that reveal just how much undiscovered country there is in the field of discrete mathematics.