1. Let $G$ be a tree on 5 vertices. Prove that $G$ can have at most one vertex with degree $\geq 3$. 
2. Find a graph $G$ such that the largest clique of $G$ and the largest independent set of $G$ are both of size 4. What can we say about the sizes of the largest clique of $\overline{G}$ and the largest independent set of $\overline{G}$?
3. Prove that for any graph $G$, either $G$ is connected, $\overline{G}$ is connected, or both.
4. Suppose a graph $G$ is connected and has an equal number of edges and vertices. Show that $G$ contains exactly one cycle.
5. Consider the graph illustrated below.

a) Show that it is four-colourable.

b) Show that after removing any edge from the graph it becomes three-colourable.