1. Show that for any positive integer $k$, $k^n$ is $O((k+1)^n)$, but $(k+1)^n$ is not $O(k^n)$.
2. Suppose we draw two cards from a well-shuffled standard deck of 52 cards without replacement. Let $A$ be the event that the two cards drawn have the same value (e.g. both 7s). Let $B$ be the event that the two cards drawn are both face cards (e.g. Q and K).

a) What is the probability of event $A$ given event $B$, $P(A|B)$?

b) What is the probability of event $B$ given event $A$, $P(B|A)$?

c) Are events $A$ and $B$ independent?
3. Suppose we roll a pair of fair, six-sided dice. Let the random variable $X$ be the absolute value of the difference between the numbers on the two dice. Compute the probability $P(X = a)$ for all positive integers $a$. 
4. Suppose that $X$ and $Y$ are independent random variables. Prove that 
$$\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y).$$
Does this expression still hold if $X$ and $Y$ are not independent? Explain.

Recall that if $T$ is a random variable with $E(T) = \mu$, then $\text{Var}(T) = E[(T - \mu)^2].$
5. In ordinary integer arithmetic, the following statement is true: if \( ab = 0 \), then \( a = 0 \) or \( b = 0 \). For modular arithmetic on \( \mathbb{Z}_n \), we get the corresponding statement: if \( a \oplus b = 0 \), then \( a = 0 \) or \( b = 0 \).

   a) Prove that this statement is true for any prime number \( n \).

   b) Show that this statement does not hold for any composite number \( n \).