Student Number (Required) ______________________

Name (Optional)________________________________

This is a closed book test. You may not refer to any resources.

This is a 50 minute test.

Please write your answers in ink. Pencil answers will be marked, but will not be reconsidered after the test papers have been returned.

The test will be marked out of 50.

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TOTAL /50
Question 1 (30 Marks)

You have won the contract to install Wi-Fi nodes along a very straight and sparsely populated stretch of road which runs due east and west across the tiny nation of Occiput. There are N houses along the road – each house is identified by its distance from the east end of the road. Each house is located right on the road, not set back from the road. Your assignment is to install Wi-Fi nodes along the road so that each house is no more than 1 kilometre from a node. You can install nodes anywhere along the road – the nodes do not have to be located at houses. You want to install as few nodes as possible.

This figure illustrates an instance of the problem and one possible solution. The black dots represent houses, the white dots represent Wi-Fi nodes, and the grey bars show the “1 km in each direction” range of each Wi-Fi node. The solution shown is not optimal.

a) (10 marks) Give a Greedy Algorithm to find an optimal (minimal) set of locations for the Wi-Fi nodes. (Hint: consider the west-most house – how far east of that house can you place the first node?)
b) **(10 marks)** Prove that the first choice your algorithm makes for a node location is correct (i.e. that there is an optimal solution that contains this location as its first location).

c) **(10 marks)** Complete the proof that your algorithm finds an optimal solution.
Question 2 (20 Marks)

You have landed a prestigious new job, hiring guards for the National Prison for Disgraced Politicians (a very crowded place). The prisoners must be guarded from 6 AM to 6 PM. There are a total of n guards, but each guard is only available for a specific time period during the day: Guard \( G_i \) will work during the interval \([s_i, f_i]\), where \( 0 \leq s_i < f_i \leq 24 \). Each guard is payed the same amount, regardless of how long their shift is. Since you are paying them out of your own salary, your goal is to hire as few guards as possible.

You may assume that there is a feasible solution – there are enough guards to cover the whole day.

(a) **(10 marks)** Give a Greedy Algorithm to find an optimal solution (i.e. minimal number of guards) subject to the constraint that there must be at least one guard on duty at all times between 6 AM and 6 PM. The total time period covered may start before 6 AM and may end after 6 PM.

(b) **(10 marks)** Explain why your algorithm would not work if there is an added constraint that each guard has a first name (Kim, Pat, Kelly, etc) and you cannot hire two guards with the same first name.